USA Mathematical Talent Search

PROBLEMS

Round 4 - Year 14 - Academic Year 2002–2003

1/4/14. The sequence of letters **TAGC** is written in succession 55 times on a strip, as shown below. The strip is to be cut into segments between letters, leaving strings of letters on each segment, which we will call words. For example, a cut after the first G, after the second T, and after the second C would yield the words **TAG**, **CT**, and **AGC**. At most how many distinct words could be found if the entire strip were cut? Justify your answer.

T A G C T A G C T A G ··· **C T A G C**

2/4/14. We define the number s as

$$s = \sum_{i=1}^{\infty} \frac{1}{10^i - 1} = \frac{1}{9} + \frac{1}{99} + \frac{1}{999} + \frac{1}{9999} + \ldots = 0.12232424\ldots .$$

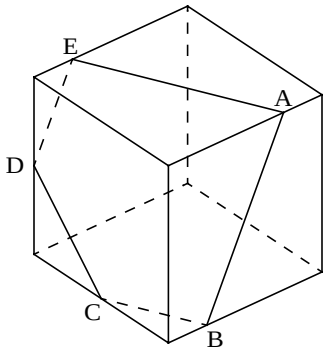
We can determine the nth digit right of the decimal point of s without summing the entire infinite series because after summing the first n terms of the series, the rest of the series sums to less than $2/10^{n+1}$. Determine the smallest prime number p for which the pth digit right of the decimal point of s is greater than 2. Justify your answer.

3/4/14. Find the real-numbered solution to the equation below and demonstrate that it is unique.

$$\frac{36}{\sqrt{x}} + \frac{9}{\sqrt{y}} = 42 - 9\sqrt{x} - \sqrt{y}$$

4/4/14. Two overlapping triangles could divide the plane into up to eight regions, and three overlapping triangles could divide the plane into up to twenty regions. Find, with proof, the maximum number of regions into which six overlapping triangles could divide the plane. Describe or draw an arrangement of six triangles that divides the plane into that many regions.

5/4/14. Prove that if the cross-section of a cube cut by a plane is a pentagon, as shown in the figure on the right, then there are two adjacent sides of the pentagon such that the sum of the lengths of those two sides is greater than the sum of the lengths of the other three sides. For ease of grading, please use the names of the points from the figure on the right in your solution.



Complete, well-written solutions to at least two of the problems above, accompanied by a **Cover Sheet,** should be mailed to:

USA Mathematical Talent Search
National Conference Services, Inc.
6440-C Dobbin Road
Columbia, MD 21045-4770

and **postmarked no later than March 16, 2003**. Each participant is expected to develop solutions without help from others. For the cover sheet and other details, see the USAMTS web site: `http://www.nsa.gov/programs/mepp/usamts.html`.