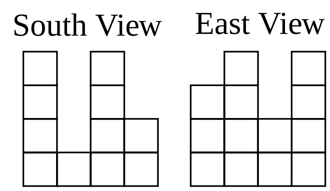
USA Mathematical Talent Search

PROBLEMS

Round 1 - Year 14 - Academic Year 2002-2003

1/1/14. Some unit cubes are stacked atop a flat 4 by 4 square. The figures show views of the stacks from two different sides. Find the maximum and minimum number of cubes that could be in the stacks. Also give top views of a maximum arrangement and a minimum arrangement with each stack marked with its height.

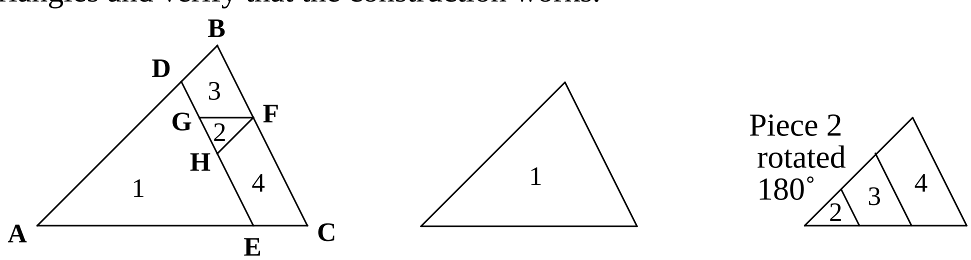


2/1/14. Find four distinct positive integers, a, b, c, and d, such that each of the four sums $a+b+c$, $a+b+d$, $a+c+d$, and $b+c+d$ is the square of an integer. Show that infinitely many quadruples (a, b, c, d) with this property can be created.

3/1/14. For a set of points in a plane, we construct the perpendicular bisectors of the line segments connecting every pair of those points and we count the number of points in which these perpendicular bisectors intersect each other. If we start with twelve points, the maximum possible number of intersection points is 1705. What is the maximum possible number of intersection points if we start with thirteen points?

4/1/14. A transposition of a vector is created by switching exactly two entries of the vector. For example, $(1, 5, 3, 4, 2, 6, 7)$ is a transposition of $(1, 2, 3, 4, 5, 6, 7)$. Find the vector X if $S = (0, 0, 1, 1, 0, 1, 1)$, $T = (0, 0, 1, 1, 1, 1, 0)$, $U = (1, 0, 1, 0, 1, 1, 0)$, and $V = (1, 1, 0, 1, 0, 1, 0)$ are all transpositions of X. Describe your method for finding X.

5/1/14. As illustrated below, we can dissect every triangle **ABC** into four pieces so that piece 1 is a triangle similar to the original triangle, while the other three pieces can be assembled into a triangle also similar to the original triangle. Determine the ratios of the sizes of the three triangles and verify that the construction works.



Complete, well-written solutions, accompanied by a **Cover Sheet** and an **Entry Form**, should be mailed to the address on the USAMTS web site

`http://www.nsa.gov/programs/mepp/usamts.html`

and **postmarked no later than 6 October 2002**. Each participant is expected to develop solutions without help from other people.