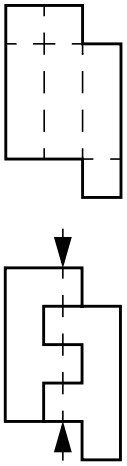
USA Mathematical Talent Search

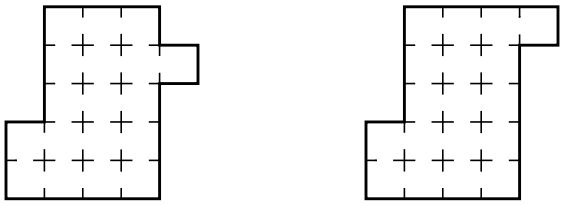
PROBLEMS
Round 4 - Year 12 - Academic Year 2000-2001

1/4/12. Determine all positive integers with the property that they are one more than the sum of the squares of their digits in base 10.

2/4/12. Prove that if n is an odd positive integer, then $N = 2269^n + 1779^n + 1730^n - 1776^n$ is an integer multiple of 2001.

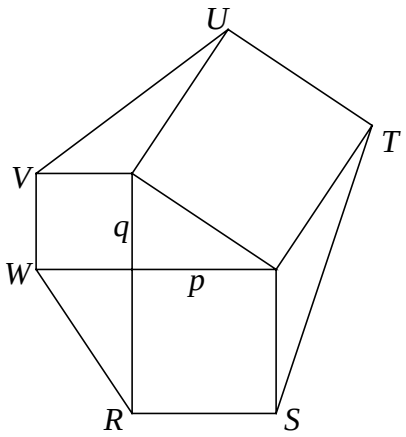
3/4/12. The figure on the right. can be divided into two congruent halves that are related to each other by a glide reflection, as shown below it. A glide reflection reflects a figure about a line, but also moves the reflected figure in a direction parallel to that line. For a square-grid figure, the only lines of reflection that keep its reflection on the grid are horizontal, vertical, 45° diagonal, and 135° diagonal. Of the two figures below, divide one figure into two congruent halves related by a glide reflection, and tell why the other figure cannot be divided like that.





4/4/12. Let A and B be points on a circle which are not diametrically opposite, and let C be the midpoint of the smaller arc between A and B. Let D, E, and F be the points determined by the intersections of the tangent lines to the circle at A, B, and C. Prove that the area of ΔDEF is greater than half of the area of ΔABC.

5/4/12. Hexagon $RSTUVW$ is constructed by starting with a right triangle of legs measuring p and q, constructing squares outwardly on the sides of this triangle, and then connecting the outer vertices of the squares, as shown in the figure on the right.



Given that p and q are integers with $p > q$, and that the area of $RSTUVW$ is 1922, determine p and q.

****

Complete, well-written solutions to at least two of the problems above, accompanied by a completed **Cover Sheet**

(available on the web site http://www.nsa.gov/programs/mepp/usamts.html), should be sent to the following address and **postmarked no later than March 12, 2001**. Also include an **Entry Form** if you have not submitted one for this academic year. Each participant is expected to develop solutions without help from others.

USA Mathematical Talent Search
COMAP Inc., Suite 210
57 Bedford Street
Lexington, MA 02420-4350