U S A Mathematical Talent Search

PROBLEMS
Round 4 - Year 11 - Academic Year 1999-2000

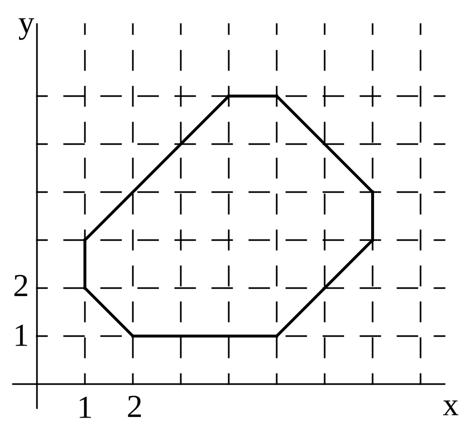
1/4/11. Determine the unique 9-digit integer M that has the following properties: (1) its digits are all distinct and nonzero; and (2) for every positive integer $m = 2, 3, 4, ..., 9$, the integer formed by the leftmost m digits of M is divisible by m.

2/4/11. The Fibonacci numbers are defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$. It is well-known that the sum of any 10 consecutive Fibonacci numbers is divisible by 11. Determine the smallest integer N so that the sum of any N consecutive Fibonacci numbers is divisible by 12.

3/4/11. Determine the value of

$$S = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \ldots + \sqrt{1 + \frac{1}{1999^2} + \frac{1}{2000^2}}$$

4/4/11. We will say that an octagon is integral if it is equiangular, its vertices are lattice points (i.e., points with integer coordinates), and its area is an integer. For example, the figure on the right shows an integral octagon of area 21. Determine, with proof, the smallest positive integer K so that for every positive integer $k \geq K$, there is an integral octagon of area k.



5/4/11. (Revised 2-4-2000) Let P be a point interior to square $ABCD$ so that $PA = a$, $PB = b$, $PC = c$, and $c^2 = a^2 + 2b^2$. Given only the lengths a, b, and c, and using only a compass and straightedge, construct a square congruent to square ABCD.

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Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet should be sent to the following address and **postmarked no later than March 13, 2000**. Each participant is expected to develop solutions without help from others.

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