U S A Mathematical Talent Search

PROBLEMS
Round 1 - Year 11 - Academic Year 1999-2000

1/1/11. The digits of the three-digit integers *a, b,* and *c* are the nine nonzero digits 1, 2, 3, ..., 9, each of them appearing exactly once. Given that the ratio *a:b:c* is 1:3:5, determine *a, b,* and *c*.

2/1/11. Let N = 111...1222...2, where there are 1999 digits of 1 followed by 1999 digits of 2. Express N as the product of four integers, each of them greater than 1.

3/1/11. Triangle *ABC* has angle *A* measuring $30°$, angle *B* measuring $60°$, and angle *C* measuring $90°$. Show four different ways to divide triangle *ABC* into four triangles, each similar to triangle ABC, but with one quarter of the area. Prove that the angles and sizes of the smaller triangles are correct.

4/1/11. There are 8436 steel balls, each with radius 1 centimeter, stacked in a tetrahedral pile, with one ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. Determine the height of the pile in centimeters.

5/1/11. In a convex pentagon ABCDE the sides have lengths 1, 2, 3, 4, and 5, though not necessarily in that order. Let F, G, H, and I be the midpoints of the sides AB, BC, CD, and DE, respectively. Let X be the midpoint of segment FH, and Y be the midpoint of segment GI. The length of segment XY is an integer. Find all possible values for the length of side AE.

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Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet and a completed Entry Form, should be sent to the following address and **postmarked no later than September 13, 1999**. Each participant is expected to develop solutions without help from others.

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