USA Mathematical Talent Search

Round 4 Problems

Year 16 — Academic Year 2004–2005

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Please follow the rules below to ensure that your paper is graded properly.

1. Put your name, username, and USAMTS ID# on **every page you submit**.
2. Once you send in your solutions, that submission is final. You cannot resubmit solutions.
3. If you have already sent in an Entry Form and a Permission Form, you do not need to resend them.
4. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging into the site, then clicking on My USAMTS on the sidebar, then click Profile. If you are registered for the USAMTS and haven't received any email from us about the USAMTS, your email address is probably wrong in your Profile.
5. Do not fax solutions written in pencil.
6. No single page should contain solutions to more than one problem.
7. Submit your solutions by March 14, 2005 (postmark deadline), via one of the methods below.
   (a) Email: solutions@usamts.org. Please see usamts.org for a list of acceptable file types. Do not send .doc Microsoft Word files.
   (b) Fax: (619) 445-2379
   (c) Snail mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090.
8. Re–read item 1.

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1/4/16. Determine with proof the number of positive integers n such that a convex regular polygon with n sides has interior angles whose measures, in degrees, are integers.

2/4/16. Find positive integers a, b, and c such that

$$\sqrt{a}+\sqrt{b}+\sqrt{c}=\sqrt{219+\sqrt{10080}+\sqrt{12600}+\sqrt{35280}}.$$

Prove that your solution is correct. (Warning: numerical approximations of the values do not constitute a proof.)

3/4/16. Find, with proof, a polynomial $f(x, y, z)$ in three variables, with integer coefficients, such that for all integers a, b, c, the sign of $f(a, b, c)$ (that is, positive, negative, or zero) is the same as the sign of $a+b\sqrt[3]{2}+c\sqrt[3]{4}$.

4/4/16. Find, with proof, all integers n such that there is a solution in nonnegative real numbers (x, y, z) to the system of equations

$$2x^2+3y^2+6z^2=n \quad \text{and} \quad 3x+4y+5z=23.$$

5/4/16. Medians AD, BE, and CF of triangle ABC meet at G as shown. Six small triangles, each with a vertex at G, are formed. We draw the circles inscribed in triangles AFG, BDG, and CDG as shown. Prove that if these three circles are all congruent, then ABC is equilateral.