USA Mathematical Talent Search
Round 1 Problems
Year 16 — Academic Year 2004–2005
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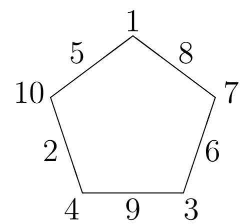
1/1/16. The numbers 1 through 10 can be arranged along the vertices and sides of a pentagon so that the sum of the three numbers along each side is the same. The diagram on the right shows an arrangement with sum 16. Find, with proof, the smallest possible value for a sum and give an example of an arrangement with that sum.



2/1/16. For the equation

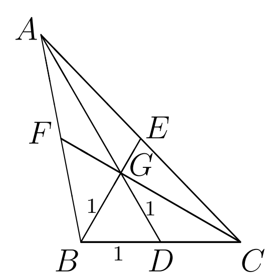
$$(3x^2 + y^2 - 4y - 17)^3 - (2x^2 + 2y^2 - 4y - 6)^3 = (x^2 - y^2 - 11)^3,$$

determine its solutions (x, y) where both x and y are integers. Prove that your answer lists all the integer solutions.

3/1/16. Given that $5r + 4s + 3t + 6u = 100$, where $r \geq s \geq t \geq u \geq 0$ are real numbers, find, with proof, the maximum and minimum possible values of $r + s + t + u$.

4/1/16. The interior angles of a convex polygon form an arithmetic progression with a common difference of 4°. Determine the number of sides of the polygon if its largest interior angle is 172°.

5/1/16. Point G is where the medians of the triangle ABC intersect and point D is the midpoint of side $\overline{BC}$. The triangle BDG is equilateral with side length l. Determine the lengths, AB, BC, and CA, of the sides of triangle ABC.



Round 1 Solutions must be submitted by **October 4, 2004**.
Please visit **http://www.usamts.org** for details about solution submission.