USA Mathematical Talent Search
Round 1 Problems
Year 29 — Academic Year 2017–2018
www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by October 25, 2017, via one (and only one!) of the methods below:

   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 8 PM Eastern / 5 PM Pacific on October 25, 2017

   (b) Mail: USAMTS
   P.O. Box 4499
   New York, NY 10163
   (Solutions must be postmarked on or before October 25, 2017.)

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the "My USAMTS" pages.

7. Round 1 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on www.usamts.org.

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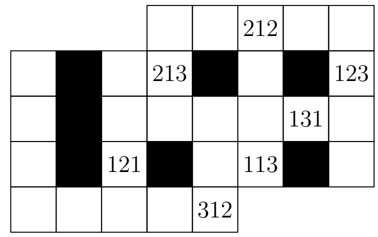
Each problem is worth 5 points.

1/1/29. Fill each white square in with a number so that each of the 27 three-digit numbers whose digits are all 1, 2, or 3 is used exactly once. For each pair of white squares sharing a side, the two numbers must have equal digits in exactly two of the three positions (ones, tens, hundreds). Some numbers have been given to you.



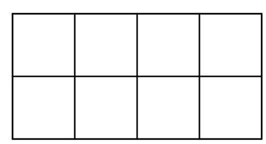
111 112 ~~113~~   211 ~~212~~ ~~213~~   311 ~~312~~ 313
~~121~~ 122 ~~123~~   221 222 223   321 322 323
~~131~~ 132 133   231 232 233   331 332 333

You do not need to prove that your answer is the only one possible; you merely need to find an answer that satisfies the constraints above. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

2/1/29. After each Goober ride, the driver rates the passenger as 1, 2, 3, 4, or 5 stars. The passenger's overall rating is determined as the average of all of the ratings given to him or her by drivers so far. Noah had been on several rides, and his rating was neither 1 nor 5. Then he got a 1 star on a ride because he barfed on the driver. Show that the number of 5 stars that Noah needs in order to climb back to at least his overall rating before barfing is independent of the number of rides that he had taken.

3/1/29. Do there exist two polygons such that, by putting them together in three different ways (without holes, overlap, or reflections), we can obtain first a triangle, then a convex quadrilateral, and lastly a convex pentagon?

4/1/29. Two players take turns placing an unused number from $\{1, 2, 3, 4, 5, 6, 7, 8\}$ into one of the empty squares in the array to the right. The game ends once all the squares are filled. The first player wins if the product of the numbers in the top row is greater. The second player wins if the product of the numbers in the bottom row is greater. If both players play with perfect strategy, who wins this game?



5/1/29. Does there exist a set S consisting of rational numbers with the following property: for every integer n there is a unique nonempty, finite subset of S, whose elements sum to n?

Problems by Nikolai Beluhov, Tristan Pollner, Billy Swartworth, and USAMTS Staff.
Round 1 Solutions must be submitted by **October 25, 2017**.
Please visit **http://www.usamts.org** for details about solution submission.