USA Mathematical Talent Search
Round 1 Problems
Year 28 — Academic Year 2016–2017
www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by **October 17, 2016**, via one (and only one!) of the methods below:

   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 8 PM Eastern / 5 PM Pacific on October 17, 2016

   (b) Mail: USAMTS
   PO Box 4499
   New York, NY 10163
   Deadline: Solutions must be postmarked on or before October 17, 2016.
   We **strongly recommend** that you keep a copy of your solutions and that you pay for tracking on the mailing. With large envelopes, there have been significant delays in the past.

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the "My USAMTS" pages.

7. Round 1 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My USAMTS." You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules at www.usamts.org.

USA Mathematical Talent Search

Round 1 Problems

Year 28 — Academic Year 2016–2017

www.usamts.org

Each problem is worth 5 points.

1/1/28. Fill in each cell of the grid with one of the numbers 1, 2, or 3. After all numbers are filled in, if a row, column, or any diagonal has a number of cells equal to a multiple of 3, then it must have the same amount of 1's, 2's, and 3's. (There are 10 such diagonals, and they are all marked in the grid by a gray dashed line.) Some numbers have been given to you.

	2	1						
3			2					
			2			3	2	
	2	1			1			3
3					3			3
2						2	3	
3	2	3	2		2			3
					3			1
						1	3	

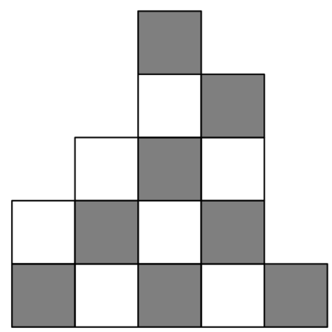
You do not need to prove that your answer is the only one possible; you merely need to find an answer that satisfies the constraints above. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

2/1/28. A tower of height h is a stack of contiguous rows of squares of height h such that

(i) the bottom row of the tower has h squares,

(ii) each row above the bottom row has one fewer square than the row below it, and within each row the squares are contiguous,

(iii) the squares in any given row all lie directly above a square in the row below.



A tower is called balanced if when the squares of the tower are colored black and white in a checkerboard fashion, the number of black squares is equal to the number of white squares. For example, the figure above shows a tower of height 5 that is not balanced, since there are 7 white squares and 8 black squares.

How many balanced towers are there of height 2016?

3/1/28. Find all positive integers n for which $(x^n + y^n + z^n)/2$ is a perfect square whenever x, y, and z are integers such that $x + y + z = 0$.

4/1/28. Find all functions $f(x)$ from nonnegative reals to nonnegative reals such that $f(f(x)) = x^4$ and $f(x) \le Cx^2$ for some constant C.

5/1/28. Let $ABCD$ be a convex quadrilateral with perimeter $\frac{5}{2}$ and $AC = BD = 1$. Determine the maximum possible area of $ABCD$.

USA Mathematical Talent Search

Round 1 Problems

Year 28 — Academic Year 2016–2017

www.usamts.org

Problems by Evan Chen, Aaron Doman, Billy Swartworth, and USAMTS Staff.

Round 1 Solutions must be submitted by **October 17, 2016**.

Please visit **http://www.usamts.org** for details about solution submission.