USA Mathematical Talent Search

Round 3 Problems

Year 26 — Academic Year 2014–2015

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by January 19, 2015, via one (and only one!) of the methods below:

   (a) Web: Log on to `www.usamts.org` to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 3 PM Eastern / Noon Pacific on January 19

   (b) Mail: **USAMTS**
   PO Box 4499
   New York, NY 10163
   (Solutions must be postmarked on or before January 19.)

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto `www.usamts.org` and visiting the "My USAMTS" pages.

7. Round 3 results will be posted at `www.usamts.org` when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on `www.usamts.org`.

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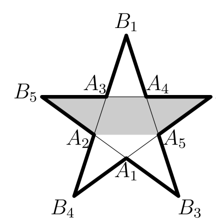
Each problem is worth 5 points.

1/3/26. Fill in each blank unshaded cell with a positive integer less than 100, such that every consecutive group of unshaded cells within a row or column is an arithmetic sequence.

You do not need to prove that your answer is the only one possible; you merely need to find an answer that satisfies the constraints above. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

3							59			
	10									
				31	26					59

2/3/26. Let $A_1A_2A_3A_4A_5$ be a regular pentagon with side length 1. The sides of the pentagon are extended to form the 10-sided polygon shown in bold at right. Find the ratio of the area of quadrilateral $A_2A_5B_2B_5$ (shaded in the picture to the right) to the area of the entire 10-sided polygon.



3/3/26. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive real numbers such that:

(i) For all positive integers m and n, we have $a_{mn} = a_m a_n$, and

(ii) There exists a positive real number B such that for all positive integers m and n with $m < n$, we have $a_m < Ba_n$.

Find all possible values of $\log_{2015}(a_{2015}) - \log_{2014}(a_{2014})$.

4/3/26. Nine distinct positive integers are arranged in a circle such that the product of any two non-adjacent numbers in the circle is a multiple of n and the product of any two adjacent numbers in the circle is not a multiple of n, where n is a fixed positive integer. Find the smallest possible value for n.

5/3/26. A finite set S of unit squares is chosen out of a large grid of unit squares. The squares of S are tiled with isosceles right triangles of hypotenuse 2 so that the triangles do not overlap each other, do not extend past S, and all of S is fully covered by the triangles. Additionally, the hypotenuse of each triangle lies along a grid line, and the vertices of the triangles lie at the corners of the squares. Show that the number of triangles must be a multiple of 4.

Round 3 Solutions must be submitted by **January 19, 2015**.
Please visit **http://www.usamts.org** for details about solution submission.