USA Mathematical Talent Search
Round 2 Problems
Year 26 — Academic Year 2014–2015
www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.
2. Put your name and USAMTS ID# on **every page you submit**.
3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.
4. Submit your solutions by December 8, 2014, via one (and only one!) of the methods below:
   (a) Web: Log on to `www.usamts.org` to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 3 PM Eastern / Noon Pacific on December 8, 2014.
   (b) Mail: **USAMTS**
   PO Box 4499
   New York, NY 10163
   (Solutions must be postmarked on or before December 8, 2014.)
5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.
6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto `www.usamts.org` and visiting the "My USAMTS" pages.
7. Round 2 results will be posted at `www.usamts.org` when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on `www.usamts.org`.

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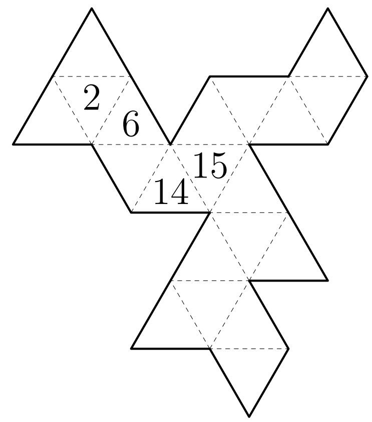
Each problem is worth 5 points.

1/2/26. The net of 20 triangles shown to the right can be folded to form a regular icosahedron. Inside each of the triangular faces, write a number from 1 to 20 with each number used exactly once. Any pair of numbers that are consecutive must be written on faces sharing an edge in the folded icosahedron, and additionally, 1 and 20 must also be on faces sharing an edge. Some numbers have been given to you.
You do not need to prove that your answer is the only one possible; you merely need to find an answer that satisfies the constraints above. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)



2/2/26. Let a, b, c, x, y be positive real numbers such that

$$ax + by \leq bx + cy \leq cx + ay.$$

Prove that $b \leq c$.

3/2/26. Let $\mathcal{P}$ be a square pyramid whose base consists of the four vertices $(0,0,0)$, $(3,0,0)$, $(3,3,0)$, and $(0,3,0)$, and whose apex is the point $(1,1,3)$. Let $\mathcal{Q}$ be a square pyramid whose base is the same as the base of $\mathcal{P}$, and whose apex is the point $(2,2,3)$. Find the volume of the intersection of the interiors of $\mathcal{P}$ and $\mathcal{Q}$.

4/2/26. A point P in the interior of a convex polyhedron in Euclidean space is called a *pivot point* of the polyhedron if every line through P contains exactly 0 or 2 vertices of the polyhedron. Determine, with proof, the maximum number of pivot points that a polyhedron can contain.

5/2/26. Find the smallest positive integer n that satisfies the following: We can color each positive integer with one of n colors such that the equation

$$w + 6x = 2y + 3z$$

has no solutions in positive integers with all of w, x, y, z the same color. (Note that w, x, y, z need not be distinct: for example, 5 and 7 must be different colors because $(w, x, y, z) = (5, 5, 7, 7)$ is a solution to the equation.)

Round 2 Solutions must be submitted by **December 8, 2014**.
Please visit **http://www.usamts.org** for details about solution submission.