USA Mathematical Talent Search

Round 1 Problems

Year 26 — Academic Year 2014–2015

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by November 5, 2014, via one (and only one!) of the methods below:

   (a) Web: Log on to `www.usamts.org` to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 3 PM Eastern / Noon Pacific on November 5, 2014.

   (b) Mail: **USAMTS**
   PO Box 4499
   New York, NY 10163
   (Solutions must be postmarked on or before November 5, 2014.)

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto `www.usamts.org` and visiting the "My USAMTS" pages.

7. Round 1 results will be posted at `www.usamts.org` when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on `www.usamts.org`.

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Each problem is worth 5 points.

1/1/26. Divide the grid shown to the right into more than one region so that the following rules are satisfied.

					6	5	6
4			2			4	
	3		3		4		
				4			
		4				3	
	4			4			4
1	1	1					

1. Each unit square lies entirely within exactly 1 region.
2. Each region is a single piece connected by the edges of its unit squares.
3. Each region contains the same number of whole unit squares.
4. Each region contains the same sum of numbers.

You do not need to prove that your configuration is the only one possible; you merely need to find a configuration that works. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

2/1/26. Find all triples (x, y, z) such that x, y, z, $x - y$, $y - z$, $x - z$ are all prime positive integers.

3/1/26. A group of people is lined up in *almost-order* if, whenever person A is to the left of person B in the line, A is not more than 8 centimeters taller than B. For example, five people with heights 160, 165, 170, 175, and 180 centimeters could line up in almost-order with heights (from left-to-right) of 160, 170, 165, 180, 175 centimeters.

(a) How many different ways are there to line up 10 people in almost-order if their heights are 140, 145, 150, 155, 160, 165, 170, 175, 180, and 185 centimeters?

(b) How many different ways are there to line up 20 people in almost-order if their heights are 120, 125, 130, 135, 140, 145, 150, 155, 160, 164, 165, 170, 175, 180, 185, 190, 195, 200, 205, and 210 centimeters? (Note that there is someone of height 164 centimeters.)

4/1/26. Let ω_P and ω_Q be two circles of radius 1, intersecting in points A and B. Let P and Q be two regular n-gons (for some positive integer $n \geq 4$) inscribed in ω_P and ω_Q, respectively, such that A and B are vertices of both P and Q. Suppose a third circle ω of radius 1 intersects P at two of its vertices C, D and intersects Q at two of its vertices E, F. Further assume that A, B, C, D, E, F are all distinct points, that A lies outside of ω, and that B lies inside ω. Show that there exists a regular $2n$-gon that contains C, D, E, F as four of its vertices.

5/1/26. Let $a_0, a_1, a_2, \ldots$ be a sequence of nonnegative integers such that $a_2 = 5$, $a_{2014} = 2015$, and $a_n = a_{a_{n-1}}$ for all positive integers n. Find all possible values of a_{2015}.

Round 1 Solutions must be submitted by **November 5, 2014**.
Please visit **www.usamts.org** for details about solution submission.