USA Mathematical Talent Search

Round 1 Problems

Year 25 — Academic Year 2013–2014

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 2 , you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by October 15, 2013, via one (and only one!) of the methods below:

   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 3 PM Eastern / Noon Pacific on October 15

   (b) Mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090.
   (Solutions must be postmarked on or before October 15.)

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto www.usamts.org and visiting the "My USAMTS" pages.

7. Round 1 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on www.usamts.org.

USA Mathematical Talent Search

Round 1 Problems

Year 25 — Academic Year 2013–2014

www.usamts.org

Each problem is worth 5 points.

1/1/25. Alex is trying to open a lock whose code is a sequence that is three letters long, with each of the letters being one of A, B or C, possibly repeated. The lock has three buttons, labeled A, B and C. When the most recent 3 button-presses form the code, the lock opens. What is the minimum number of total button presses Alex needs to guarantee opening the lock?

2/1/25. In the 5×6 grid shown, fill in all of the grid cells with the digits 0–9 so that the following conditions are satisfied:

				7	
	8				6
		2	4		
5				1	
	3				

1. Each digit gets used exactly 3 times.
2. No digit is greater than the digit directly above it.
3. In any four cells that form a 2×2 subgrid, the sum of the four digits must be a multiple of 3.

You do not need to prove that your configuration is the only one possible; you merely need to find a configuration that works. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

3/1/25. An infinite sequence of positive real numbers $a_1, a_2, a_3, \ldots$ is called *territorial* if for all positive integers i, j with $i < j$, we have $|a_i - a_j| \geq \frac{1}{j}$. Can we find a territorial sequence $a_1, a_2, a_3, \ldots$ for which there exists a real number c with $a_i < c$ for all i?

4/1/25. Bunbury the bunny is hopping on the positive integers. First, he is told a positive integer n. Then Bunbury chooses positive integers a, d and hops on all of the spaces $a, a+d, a+2d, \ldots, a+2013d$. However, Bunbury must make these choices so that the number of every space that he hops on is less than n and relatively prime to n.

A positive integer n is called *bunny-unfriendly* if, when given that n, Bunbury is unable to find positive integers a, d that allow him to perform the hops he wants. Find the maximum bunny-unfriendly integer, or prove that no such maximum exists.

5/1/25. Niki and Kyle play a triangle game. Niki first draws $\triangle ABC$ with area 1, and Kyle picks a point X inside $\triangle ABC$. Niki then draws segments $\overline{DG}$, $\overline{EH}$, and $\overline{FI}$, all through X, such that D and E are on $\overline{BC}$, F and G are on $\overline{AC}$, and H and I are on $\overline{AB}$. The ten points must all be distinct. Finally, let S be the sum of the areas of triangles DEX, FGX, and HIX. Kyle earns S points, and Niki earns $1 - S$ points. If both players play optimally to maximize the amount of points they get, who will win and by how much?

Round 1 Solutions must be submitted by **October 15, 2013**.
Please visit **http://www.usamts.org** for details about solution submission.