USA Mathematical Talent Search
Round 2 Problems
Year 23 — Academic Year 2011–2012
`www.usamts.org`

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer for a problem with no proof, you will get no more than 1 point.

2. Put your name and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page, and you should only submit work on one side of each piece of paper.

4. Submit your solutions by Monday, November 14, 2011, via one (and only one!) of the methods below:

   (a) Web: Log on to `www.usamts.org` to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 3 PM Eastern / Noon Pacific on November 14, 2011.

   (b) Mail: USAMTS, P.O. Box 2090, Alpine, CA 91903–2090.
   (Solutions must be postmarked on or before November 14.)

5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.

6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging onto `www.usamts.org` and visiting the "My USAMTS" pages.

7. Round 2 results will be posted at `www.usamts.org` when available. To see your results, log on to the USAMTS website, then go to "My USAMTS". You will also receive an email when your scores and comments are available (provided that you did item #6 above).

These are only part of the complete rules.
Please read the entire rules on `www.usamts.org`.

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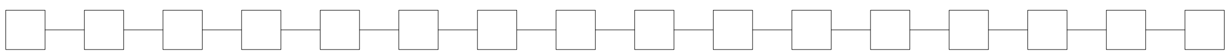
Each problem is worth 5 points.

1/2/23. Find all the ways of placing the integers 1, 2, 3, . . . , 16 in the boxes below, such that each integer appears in exactly one box, and the sum of every pair of neighboring integers is a perfect square.



2/2/23. Four siblings are sitting down to eat some mashed potatoes for lunch: Ethan has 1 ounce of mashed potatoes, Macey has 2 ounces, Liana has 4 ounces, and Samuel has 8 ounces. This is not fair. A *blend* consists of choosing any two children at random, combining their plates of mashed potatoes, and then giving each of those two children half of the combination. After the children's father performs four blends consecutively, what is the probability that the four children will all have the same amount of mashed potatoes?

3/2/23. Find all integers b such that there exists a positive real number x with

$$\frac{1}{b} = \frac{1}{\lfloor 2x \rfloor} + \frac{1}{\lfloor 5x \rfloor}.$$

Here $\lfloor y \rfloor$ denotes the greatest integer that is less than or equal to y.

4/2/23. A *luns* with vertices X and Y is a region bounded by two circular arcs meeting at the endpoints X and Y. Let A, B, and V be points such that $\angle AVB = 75^\circ$, $AV = \sqrt{2}$ and $BV = \sqrt{3}$. Let L be the largest area luns with vertices A and B that does not intersect the lines $\overleftrightarrow{VA}$ or $\overleftrightarrow{VB}$ in any points other than A and B. Define k as the area of L. Find the value

$$\frac{k}{(1+\sqrt{3})^2}.$$

5/2/23. Miss Levans has 169 students in her history class and wants to seat them all in a 13×13 grid of desks. Each desk is placed at a different vertex of a 12 meter by 12 meter square grid of points she has marked on the floor. The distance between neighboring vertices is exactly 1 meter.

Each student has at most three best friends in the class. Best-friendship is mutual: if Lisa is one of Shannon's best friends, then Shannon is also one of Lisa's best friends. Miss Levans knows that if any two best friends sit at points that are 3 meters or less from each other then they will be disruptive and nobody will learn any history. And that is bad.

Prove that Miss Levans can indeed place all 169 students in her class without any such disruptive pairs.

Round 2 Solutions must be submitted by **November 14, 2011**.
Please visit **http://www.usamts.org** for details about solution submission.