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**3/3/19.** Consider all polynomials $f(x)$ with integer coefficients such that $f(200) = f(7) = 2007$ and $0 < f(0) < 2007$. Show that the value of $f(0)$ does not depend on the choice of polynomial, and find $f(0)$.

**Comments** This problem may be solved by using the following crucial result: If $p(x)$ is a polynomial with integer coefficients, then for any integers $a$ and $b$, $p(a) - p(b)$ is a multiple of $a - b$. In particular, for the case $b = 0$, $p(a) - p(0)$ is a multiple of $a$. *Solutions edited by Naoki Sato.*

**Solution by: Andy Zhu (11/NJ)**

Since all polynomials $P(x)$ with integer coefficients can be expressed in the form $P(x) = x \cdot Q(x) + P(0)$, where $Q(x)$ is a polynomial with integer coefficients, $P(n) \equiv P(0) \pmod{n}$ for all positive integers $n$. Thus $f(0) \equiv f(7) \equiv 2007 \pmod{7}$ and $f(0) \equiv f(200) \equiv 2007 \pmod{200}$.

Since $\gcd\{7, 200\} = 1$, we can apply the Chinese Remainder Theorem to get $f(0) \equiv 2007 \equiv 607 \pmod{1400}$. The unique value which $f(0)$ takes in the range $0 < f(0) < 2007$ is 607.