2/2/18. $ABCD$ is a trapezoid with $\overline{BC} \parallel \overline{AD}$, $\angle ADC =$ 57°, $\angle DAB = 33°$, $BC = 6$, and $AD = 10$. M and N are the midpoints of $\overline{BC}$ and $\overline{AD}$, respectively.
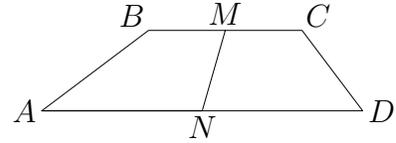

(a) Find $\angle MNA$.

(b) Find MN.

Credit This problem was proposed by Gregory Galperin.

Comments The fact that the angles 57° and 33° are complementary is a strong indication to "complete" the right triangle, as in the solution below. Once this right triangle is constructed, the required data is easy to calculate. The fact that lines AB, CD and MN are concurrent must be justified. *Solutions edited by Naoki Sato.*

Solution 1 by: David Corwin (10/MA)

Extend AB and DC to meet at point E. Because $BC \parallel AD$, segments BC and AD are homothetic with respect to point E, with ratio $\frac{AD}{BC} = \frac{10}{6} = \frac{5}{3}$. Because M is the midpoint of BC, its corresponding homothetic point on AD is the midpoint of AD, which is N, so M and N are homothetic with respect to point E, and therefore E, M, and N are collinear.
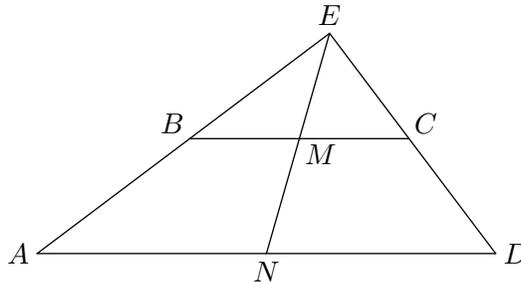
(a) By triangle AED, $\angle AED = 180° - \angle EAD - \angle EDA = 180° - 33° - 57° = 90°$, so triangle AED is right. Because N is the midpoint of hypotenuse AD, $NE = AN$, so triangle ANE is isosceles, and $\angle NEA = \angle NAE = 33°$. Then by triangle ANE, $\angle ANE = \angle ANM = 180° - \angle NEA - \angle NAE = 180° - 33° - 33° = 114°$.

(b) Because N is the midpoint of AD, $NE = AN = \frac{AD}{2} = \frac{10}{2} = 5$. By the homothety, $ME = \frac{NE}{\frac{5}{3}} = \frac{5}{\frac{5}{3}} = 3$. Then $MN = NE - ME = 5 - 3 = 2$.